Int J Theor Phys (2008) 47: 2543–2553
DOI 10.1007/s10773-008-9688-z

Evolution of Bouncing Cosmological Models Due to the Self-Gravitational Corrections

M.R. Setare

Received: 17 November 2007 / Accepted: 8 February 2008 / Published online: 15 February 2008

Abstract A four-dimensional timelike brane with non-zero energy density is considered as the boundary of a five dimensional Schwarzschild anti de Sitter bulk background. The self-gravitational corrections to the first Friedmann equation act as a source of stiff matter contrary to standard FRW cosmology where the charge of the black hole plays this role. In a previous related paper (Setare in Eur. Phys. J. C 47:851, 2006), bouncing cosmology was studied, from a holographic perspective, for the very special case of a brane that is void of any intrinsic matter sources. In this paper we extend the results of (Setare in Eur. Phys. J. C 47:851, 2006). We consider the physically relevant case in which a perfect fluid with equation of state of radiation is present on the brane. Then, we describe solutions of the braneworld theory under investigation and also determine their stability. Specifically, if we do not consider the self-gravitational corrections, the AdS black hole with zero ADM mass, and open horizon is an attractor, while, if we consider, the AdS black hole with zero ADM mass and flat horizon, and $D3$-brane with non-zero energy density is a repeller.

Keywords Timelike brane · Self-gravitational corrections · Bouncing cosmology · Friedmann equation · Stability

1 Introduction

Motivated by string/M theory, the AdS/CFT correspondence, and the hierarchy problem of particle physics, braneworld models were studied actively in recent years [1–6]. In these models, our universe is realized as a boundary of a higher dimensional spacetime. In particular, a well studied example is when the bulk is an AdS space. In the cosmological context, embedding of a four dimensional Friedmann-Robertson-Walker universe was also considered when the bulk is described by AdS or AdS black hole [7–33]. In the latter case, the mass of the black hole was found to effectively act as an energy density on the brane with

M.R. Setare (✉)
Department of Science, Payame Noor University, Bijar, Iran
e-mail: rezakord@ipm.ir

the same equation of state of radiation. Representing radiation as conformal matter and exploiting AdS/CFT correspondence, the Cardy-Verlinde formula [34] for the entropy was found for the universe (see [35–38], for the entropy formula in the case of dS black hole).

In either of the above cases, however, the cosmological evolution on the brane is modified at small scales. In particular, if the bulk space is taken to be an AdS black hole *with charge*, the universe can 'bounce' [39]. That is, the brane makes a smooth transition from a contracting phase to an expanding phase. From a four-dimensional point of view, singularity theorems [40] suggest that such a bounce cannot occur as long as certain energy conditions apply. Hence, a key ingredient in producing the bounce is the fact that the bulk geometry may contribute a negative energy density to the effective stress-energy on the brane [41]. At first sight these bouncing braneworlds are quite remarkable, since they provide a context in which the evolution evades any cosmological singularities while the dynamics is still controlled by a simple (orthodox) effective action. In particular, it seems that one can perform reliable calculations without deliberation on the effects of quantum gravity or the details of the ultimate underlying theory. Hence, several authors [42–47, 49] have pursued further developments for these bouncing braneworlds. However the authors of [51] have found that generically these cosmologies are in fact singular. In particular, they have shown that a bouncing brane must cross the Cauchy horizon in the bulk space. However, the latter surface is unstable when arbitrarily small excitations are introduced in the bulk spacetime.

Black hole thermodynamic quantities depend on the Hawking temperature via the usual thermodynamic relations. The Hawking temperature undergoes corrections from many sources: the quantum corrections [52–57], the self-gravitational corrections [58–63], and the corrections due to the generalized uncertainty principle [64–66]. Concerning the quantum process called Hawking effect [67] much work has been done using a fixed background during the emission process. The idea of Keski-Vakkuri, Kraus and Wilczek (KKW) [58–60] is to view the black hole background as dynamical by treating the Hawking radiation as a tunnelling process. The energy conservation is the key to this description. The total (ADM) mass is kept fixed while the mass of the black hole under consideration decreases due to the emitted radiation. The effect of this modification gives rise to additional terms in the formulae concerning the known results for black holes [61–63]; a non-thermal partner to the thermal spectrum of the Hawking radiation shows up.

In this paper we take into account corrections to the entropy of the five-dimensional Schwarzschild- anti de Sitter black hole (abbreviated to $SAdS_5$ in the sequel) that arise due to the self-gravitational effect. The self-gravitational correction, acts as a source for stiff matter on the brane, whose equation of state is simply given by the pressure being equal to the energy density. Due to the self-gravitational corrections, a bouncing universe could, arise [48, 49]. In a previous paper [50] we have considered an empty critical brane. The effective cosmological constant Λ_4 on the brane gets a contribution from the bulk cosmological constant and the brane tension. For the critical brane we finely tune Λ_4 to zero. In the non-critical empty brane case, the solution of FRW equation at very small value of the scale factor, are close to the behaviour for critical brane case. The cosmological constant term dominant at large value of scale factor, where the stiff matter term becomes irrelevant. In this paper we focus on the critical non-empty brane case. This is more physically relevant case in which a perfect fluid with equation of state of radiation is present on the brane. The first Friedmann equation gets a contribution proportional to a^{-6} due to the self-gravitational corrections with an unconventional negative sign. This term behave like the stiff matter on the brane. It is obvious that it is dominant at early time of the brane evolution, also have interesting cosmological consequences. The bounce can be attributed to the negative-energy matter, which dominates at small values of a and create a significant enough repulsive force so that a big crunch is avoided. An interesting feature of this new model is that a bounce is no longer guaranteed but, rather, depends on satisfying a condition between the parameters which appear in the modified Hubble equation. This is somewhat different than the case of an empty brane. That is, for an empty brane, a bounce will be obtained for any non-vanishing value of the self-gravitational corrections, regardless of how small. Note that the difference between the two models can be attributed to the presence, or lack thereof, of a positive term in the Friedmann equation that goes as a^{-8}.

Then we describe solutions of the bouncing braneworld theory and also determine their stability. To do this, we use a set of convenient phase-space variables similar to those introduced in [68–70]. The critical points of the system of differential equations in the space of these variables describe interesting non-static solutions. A method for evaluating the eigenvalues of the critical points of the Friedmann and Bianchi models was introduced by Goliath and Ellis [70] and further used in the analysis by Campos and Sopuerta [68, 69] of the Randall Sundrum braneworld theory. These latter authors gave a complete description of stationary points in an appropriately chosen phase space of the cosmological setup and investigated their stability with respect to homogeneous and isotropic perturbations. Cosmological solutions and their stability with respect to homogeneous and isotropic perturbations in the braneworld model with the scalar-curvature term in the action for the brane have further been studied by Iakubovskyi and Shtanov [71].

2 Self-Gravitational Corrections to FRW Brane Cosmology

In this section we review the results of [49] briefly. In the asymptotic coordinates, the $SAdS_5$ black hole metric is

$$ds^2 = -F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2 d\Omega^2_{(3)}, \tag{1}$$

where

$$F(r) = 1 - \frac{\mu}{r^2} + r^2, \tag{2}$$

and we work in units where the AdS radius $l = 1$. The parameter μ is proportional to the ADM mass M of the black hole.

Due to the self-gravitational corrections, the modified Cardy-Verlinde formula for the entropy of the $SAdS_5$ black hole is given as [48]

$$S_{\mathrm{CFT}} = \frac{2\pi r}{3}\sqrt{\left|\left(E_C - \frac{\omega}{r}\right)\left[(2E_4 - E_C) - \frac{\omega}{r}\right]\right|} \tag{3}$$

and keeping terms up to first order in the emitted energy ω, it takes the form

$$S_{\mathrm{CFT}} = s_{\mathrm{CFT}}(1 - \epsilon\omega), \tag{4}$$

where s_{CFT} is the standard CFT entropy, and the small parameter ϵ is given by

$$\epsilon = \frac{E_4}{rE_C(2E_4 - E_C)} \tag{5}$$

where E_C is the Casimir energy, the four-dimensional energy E_4, is given by [34]

$$E_4 = \frac{l}{r}E \tag{6}$$

where E is the thermodynamical energy of the black hole.

We now consider a 4-dimensional brane in the $SAdS_5$ black hole background. This 4-dimensional brane can be regarded as the boundary of the 5-dimensional $SAdS_5$ bulk background. Let us first replace the radial coordinate r with a and so the line element (1) is rewritten as

$$ds^2 = -F(a)dt^2 + \frac{1}{F(a)}da^2 + a^2 d\Omega^2_{(3)}. \tag{7}$$

The brane is the following embedding in the bulk geometry

$$t = t(\tau), \qquad a = a(\tau) \tag{8}$$

where,

$$-F(a)\left(\frac{\partial t}{\partial \tau}\right)^2 + \frac{1}{F(a)}\left(\frac{\partial a}{\partial \tau}\right)^2 = -1. \tag{9}$$

This ensures that the brane metric is FRW,

$$ds^2_{(4)} = -d\tau^2 + a^2(\tau)d\Omega^2_{(3)}. \tag{10}$$

Thus, the 4-dimensional FRW equation describes the motion of the brane universe in the $SAdS_5$ background. It is easy to see that the matter on the brane can be regarded as radiation and consequently, the field theory on the brane should be a CFT.

Within the context of the AdS/CFT correspondence, Savonije and Verlinde studied the CFT/FRW-cosmology relation from the Randall-Sundrum type braneworld perspective [72]. They showed that the entropy formulas of the CFT coincides with the Friedmann equations when the brane crosses the black hole horizon.

In the case of a 4-dimensional timelike one of the identifications that supports the CFT/FRW-cosmology relation is [72]

$$H^2 = \left(\frac{2G_4}{V}\right)^2 S^2 \tag{11}$$

where H is the Hubble parameter defined by $H = \frac{1}{a}\frac{da}{d\tau}$ and V is the volume of the 3-sphere ($V = a^3 V_3$), and S is the entropy of the black hole. The 4-dimensional Newton constant G_4 is related to the 5-dimensional one G_5 by

$$G_4 = \frac{2}{l}G_5. \tag{12}$$

It was shown that at the moment that the 4-dimensional timelike brane crosses the cosmological horizon, i.e. when $a = a_b$, the CFT entropy and the entropy of the $SAdS_5$ black hole are identical. By substituting (4) into (11), we obtain the self-gravitational corrections to the motion of the CFT-dominated brane

$$H^2 = \left(\frac{2G_4}{V}\right)^2 s^2_{\text{CFT}}(1-\epsilon\omega)^2. \tag{13}$$

It is obvious that from the first term on the right-hand side of (13) we get the standard Friedmann equation with the appropriate normalization

$$H^2 = \frac{-k}{a^2} + \frac{8\pi G_4}{3}\rho, \tag{14}$$

ρ is the energy density defined by $\rho = E_4/V$, and k taking values $+1$, 0, -1 in order to describe, respectively, the elliptic, flat, and hyperbolic horizon geometry of the $SAdS_5$ bulk black hole. If we consider the physically relevant case in which a perfect fluid with equation of state of radiation is present on the brane, then the first Friedmann equation takes the following form [48]

$$H^2 = \frac{-k}{a^2} + \frac{8\pi G_4}{3}\rho - \frac{1}{l^2} + \frac{4\pi}{3M_p^2\rho_0}(\rho_0 + \rho_{br})^2 \tag{15}$$

where ρ_0 is the tension of the brane, while ρ_{br} is the energy density of radiation. The Hubble equation can be rewritten as

$$H^2 = \frac{-k}{a^2} + \frac{8\pi G_4}{3}\rho + \frac{\Lambda_4}{3} + \frac{8\pi}{3M_p^2}\left(\frac{\rho_{br}^2}{2\rho_0} + \rho_{br}\right) \tag{16}$$

where

$$\Lambda_4 = \frac{4\pi\rho_0}{M_p^2} - \frac{3}{l^2} = \Lambda_{br} - \frac{3}{l^2} \tag{17}$$

is the effective cosmological constant of the brane. The modified Hubble equation due to the self-gravitation correction is as

$$H^2 = \frac{-k}{a^2} + \frac{8\pi G_4}{3}\rho + \frac{\Lambda_4}{3} + \frac{8\pi}{3M_p^2}\left(\frac{\rho_{br}^2}{2\rho_0} + \rho_{br}\right) - \frac{8\pi G_4}{3}\left[\frac{4\pi G_4}{3}\frac{1}{a^2 V_3}\rho\right]\omega \tag{18}$$

where the volume V is given by $a_b^3 V_3$. As one can see from (18) the first Friedmann equation gets a contribution proportional to a^{-6} due to the self-gravitational corrections with an unconventional negative sign. This term behave like the stiff matter on the brane. It is obvious that it is dominant at early time of the brane evolution, also have interesting cosmological consequences. The bounce can be attributed to the negative-energy matter, which dominates at small values of a and create a significant enough repulsive force so that a big crunch is avoided. By tuning the bulk cosmological constant and the brane tension Λ_{br}, the effective four dimensional cosmological constant Λ_4 can be set to zero, here we would like consider this critical brane. The radiation energy density ρ_{br} on the brane is as

$$\rho_{br} = \frac{\rho_r}{a^4} \tag{19}$$

where ρ_r is a constant, then we can rewrite the cosmological equation (18) as

$$H^2 = \frac{-k}{a^2} + \left(\epsilon_3 M + \frac{8\pi\rho_r}{3M_p^2}\right)\frac{1}{a^4} - \frac{\epsilon_3^2 M\omega}{2a^6} + \frac{4\pi\rho_r^2}{3M_p^2\rho_0 a^8} \tag{20}$$

where

$$\epsilon_3 = \frac{16\pi G_5}{3V_3}. \tag{21}$$

By defining

$$A = \left(\epsilon_3 M + \frac{8\pi\rho_r}{3M_p^2}\right), \tag{22}$$

$$B = \frac{\epsilon_3^2 M\omega}{2}, \tag{23}$$

and

$$C = \frac{4\pi\rho_r^2}{3M_p^2\rho_0}. \tag{24}$$

If we compare the (20) with the modified Hubble equation of [50], we see that the second term in the parentheses in the right-hand side of this equation and also the last term which is proportional to a^{-8} are new terms. These terms are proportional to ρ_r and ρ_r^2 respectively. Therefore, if we consider a perfect fluid with the equation of state of radiation on the brane, then the Hubble equation is modified as (20). On the other hand in the empty brane case, $C = 0$ and $A \to A' = \epsilon_3 M$. In the empty brane case, a bounce will be obtained due to the self-gravitational corrections without any condition, but in the non-empty case the bounce can only occur if the parameters A, B and C satisfies the condition $B^2 \geq 4AC$.

The (20) in term of the above parameters is as following

$$H^2 = \frac{-k}{a^2} + \frac{A}{a^4} - \frac{B}{a^6} + \frac{C}{a^8}. \tag{25}$$

3 Stability of the Bouncing Solutions

In this section, we describe solutions of the braneworld theory under investigation and also determine their stability. To do this, we use a set of convenient phase-space variables similar to those introduced in [68–70]. Now, we introduce the notation similar to those of [68, 69]

$$\Omega_k = \frac{-k}{a^2H^2} = \frac{-k}{\dot{a}^2}, \qquad \Omega_A = \frac{A}{a^4H^2}, \qquad \Omega_B = \frac{-B}{a^6H^2}, \qquad \Omega_C = \frac{C}{a^8H^2} \tag{26}$$

and work in the 4-dimensional Ω-space $(\Omega_k, \Omega_A, \Omega_B, \Omega_C)$. In this space, the Ω parameters are not independent since the Friedmann equation (25) reads

$$\Omega_k + \Omega_A + \Omega_B + \Omega_C = 1. \tag{27}$$

In this case the state space defined by the variables $(\Omega_k, \Omega_A, \Omega_B, \Omega_C)$ is no longer compact (because now $\Omega_B < 0$). However, we can introduce another set of variables describing a compact state space. Firstly, instead of using the Hubble function H we will use the following quantity

$$D = \sqrt{H^2 + \frac{B}{a^6}}, \tag{28}$$

and from it, let us define the following dimensionless variables

$$Z = \frac{H}{D}, \tag{29}$$

$$\tilde{\Omega}_k = \frac{-k}{a^2D^2}, \tag{30}$$

$$\tilde{\Omega}_A = \frac{A}{a^4D^2}, \tag{31}$$

$$\tilde{\Omega}_C = \frac{C}{a^8D^2}. \tag{32}$$

From these definitions we see that now the case $H=0$ is included. Moreover, the Friedmann equation takes the following form

$$\tilde{\Omega}_k + \tilde{\Omega}_A + \tilde{\Omega}_C = 1, \tag{33}$$

which, together with the fact that $-1 \le Z \le 1$ (see (28)), implies that the state space defined by the new variables is indeed compact.

Introducing the primed time derivative

$$' = \frac{1}{D}\frac{d}{dt}, \tag{34}$$

one obtains the system of first-order differential equations

$$\begin{aligned}
D' &= -ZD[1 + (q - 2\Omega_B)Z^2],\\
Z' &= -Z^2[q - (q - 2\Omega_B)Z^2],\\
\tilde{\Omega}'_k &= 2\tilde{\Omega}_k(q - 2\Omega_B)Z^3,\\
\tilde{\Omega}'_A &= 2\tilde{\Omega}_A[-1 + (q - 2\Omega_B)Z^2]Z,\\
\tilde{\Omega}'_C &= 2\tilde{\Omega}_C[-3 + (q - 2\Omega_B)Z^2]Z,
\end{aligned} \tag{35}$$

where

$$1 + (q-2)Z^2 = 4\tilde{\Omega}_C - \tilde{\Omega}_k. \tag{36}$$

The evolution equation for D is not coupled to the rest, so we will not consider it for the dynamical study. Therefore, we just study the dynamical system for the variables $\tilde{\Omega} \equiv (Z, \tilde{\Omega}_k, \tilde{\Omega}_A, \tilde{\Omega}_C)$, determined by (35). The behavior of this system of equations in the neighborhood of its stationary point is determined by the corresponding matrix of its linearization. The real parts of its eigenvalues tell us whether the corresponding cosmological solution is stable or unstable with respect to the homogeneous perturbations [71]. To begin with, we have to find the critical points of this dynamical system, which can be written in vector form as follows

$$\Omega' = f(\Omega), \tag{37}$$

where f can be extracted from (35). The critical points, Ω^*, namely the points at which the system will stay provided it is initially at there, are given by the condition

$$f(\Omega^*) = 0. \tag{38}$$

Their dynamical character is determined by the eigenvalues of the matrix

$$\left.\frac{\partial f}{\partial \Omega}\right|_{\Omega=\Omega^*}. \tag{39}$$

If the real part of the eigenvalues of a critical point is not zero, the point is said to be *hyperbolic* [68, 69]. In this case, the dynamical character of the critical point is determined by the sign of the real part of the eigenvalues: If all of them are positive, the point is said to be a *repeller*, because arbitrarily small deviations from this point will move the system away from this state. If all of them are negative the point is called an *attractor* because if we move the system slightly from this point in an arbitrary way, it will return to it. Otherwise, we say the critical point is a *saddle* point.

We construct our models as follows:

(1) The model K_ϵ, or $(Z, \tilde{\Omega}_k, \tilde{\Omega}_A, \tilde{\Omega}_C) = (\epsilon, 1, 0, 0)$, where $\epsilon \equiv \mathrm{sgn}(Z)$. We have

$$q = 2 - \frac{2}{Z^2} = 0, \tag{40}$$

and the eigenvalues are

$$(\lambda_Z, \lambda_k, \lambda_A, \lambda_C) = (0, 0, -4Z, -12Z) = -\epsilon(0, 0, 4, 12) \tag{41}$$

(2) The model A_ϵ, or $(Z, \tilde{\Omega}_k, \tilde{\Omega}_A, \tilde{\Omega}_C) = (\epsilon, 0, 1, 0)$. We have

$$q = 2 - \frac{1}{Z^2} = 1, \tag{42}$$

and the eigenvalues are

$$(\lambda_Z, \lambda_k, \lambda_A, \lambda_C) = (2Z, 2Z, 0, -4Z) = \epsilon(2, 2, 0, -4) \tag{43}$$

(3) The model C_ϵ, or $(Z, \tilde{\Omega}_k, \tilde{\Omega}_A, \tilde{\Omega}_C) = (\epsilon, 0, 0, 1)$. We have

$$q = 2 + \frac{3}{Z^2} = 5, \tag{44}$$

and the eigenvalues are

$$(\lambda_Z, \lambda_k, \lambda_A, \lambda_C) = (10Z, 10Z, 8Z, 4Z) = \epsilon(10, 10, 8, 4) \tag{45}$$

(4) The model O, or

$$(Z, \tilde{\Omega}_k, \tilde{\Omega}_A, \tilde{\Omega}_C) = (0, \tilde{\Omega}^*_k, \tilde{\Omega}^*_A, \tilde{\Omega}^*_C)$$

where $\tilde{\Omega}^*_k$, $\tilde{\Omega}^*_A$ and $\tilde{\Omega}^*_C$ are constants satisfying (33) and (38). The eigenvalues of these points can be obtained in a straightforward manner and show a saddle point, which has not been included here. Hence, O represents a set of infinite saddle points whose line element is that of an open universe ($k = -1$) with $H = 0$.

The dynamical system (35) has three type hyperbolic critical points as follows:

(i) The model K ($k = -1$),

$$A = B = C = 0, \qquad a(t) = t,$$

with the critical point of an *attractor* type.

(ii) The model A,

$$k = B = C = 0, \qquad a(t) = (A)^{1/4}\sqrt{2t},$$

with the critical point of a *saddle point* type.

(iii) The model C,

$$k = A = B = 0, \qquad a(t) = (4\sqrt{C}t)^{1/4},$$

with the critical point of a *repeller* type.

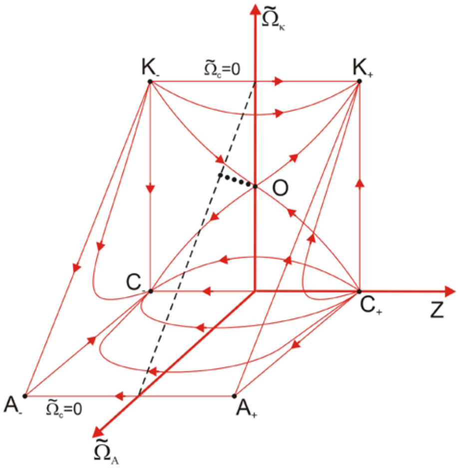

Fig. 1 State space for the bouncing braneworld models. The points k_+, A_+ and C_+ describe the critical points of an *attractor*, a *saddle point* and a *repeller*, respectively. The points k_-, A_- and C_- describe the critical points of a *repeller*, a *saddle point* and an *attractor*, respectively. The points O represent a set of infinite saddle points. Only trajectories on the invariant planes ($\tilde{\Omega}_k = 0$, $\tilde{\Omega}_A = 0$, and $\tilde{\Omega}_C = 0$), which outline the whole dynamics, are drawn

(iv) The model O,

$$(Z = 0, \tilde{\Omega}_k^*, \tilde{\Omega}_A^*, \tilde{\Omega}_C^*),$$

with the critical point of a *saddle point* type.

The four models have been depicted within the compact state space, in Fig. 1. There are just trajectories on the planes, which are invariant submanifolds of the state space.

4 Conclusion

In this paper we have considered a four-dimensional timelike brane with non-zero energy density as the boundary of the $SAdS_5$ bulk background. Exploiting the CFT/FRW-cosmology relation, we have considered the self-gravitational corrections to the first Friedmann-like equation which is the equation of the brane motion. The additional term that arises due to the semiclassical analysis, can be viewed as stiff matter where the self-gravitational corrections act as the source for it. The sign of this term is opposite with respect to the standard situation, one may expect that this sign difference could have interesting cosmological consequences. Indeed, it is crucial in allowing a nonsingular transition between a contracting and an expanding evolution of the scale factor a. This result is contrary to standard analysis that regards the charge of $SAdS_5$ bulk black hole as the source for stiff matter. Then, we have studied bouncing cosmological solutions and their stability with respect to homogeneous and isotropic perturbations in a braneworld theory. The effects of the self-gravitational corrections of five-dimensional black hole in the bulk have been considered. By including this effect in the analysis we have obtained four models with the critical points of an *attractor*, a couple *saddle points* and a *repeller*, respectively, and constructed the complete state space for these cosmological models. Specifically, if we do not consider the self-gravitational corrections, the AdS black hole with zero ADM mass, and open horizon is an attractor, while, if we consider, the AdS black hole with zero ADM mass and flat horizon, and $D3$-brane with non-zero energy density is a repeller.

References

1. Horava, P., Witten, E.: Nucl. Phys. B **460**, 460 (1996)
2. Maldacena, L.: Adv. Theor. Math. Phys. **2**, 231 (1998)
3. Antoniadis, I., Arkani-Hamed, N., Dimopoulos, S., Dvali, G.: Phys. Lett. B **436**, 257 (1998)
4. Arkani-Hamed, N., Dimopoulos, S., Dvali, G.: Phys. Lett. B **436**, 263 (1998)
5. Randall, L., Sundrum, R.: Phys. Rev. Lett. **83**, 3370 (1999)
6. Randall, L., Sundrum, R.: Phys. Rev. Lett. **83**, 4690 (1999)
7. Nihei, T.: Phys. Lett. B **465**, 81 (1999)
8. Csaki, C., Graesser, M., Kolda, C., Terning, J.: Phys. Lett. B **426**, 34 (1999)
9. Kaloper, N.: Phys. Rev. D **60**, 123506 (1999)
10. Kaloper, N., Linde, A.D.: Phys. Rev. D **59**, 101303 (1999)
11. Binetruy, P., Deffayet, C., Ellwanger, U., Langlois, D.: Phys. Lett. B **477**, 285 (2000)
12. Nojiri, S., Odintsov, S.D.: Phys. Lett. B **484**, 119 (2000)
13. Nojiri, S., Odintsov, S.D., Zerbini, S.: Phys. Rev. D **62**, 064006 (2000)
14. Anchordoqui, L., Nunez, C., Olsen, K.: J. High Energy Phys. **0010**, 050 (2000)
15. Hawking, S.W., Hertog, T., Reall, H.S.: Phys. Rev. D **62**, 043501 (2000)
16. Coule, D.H.: Class. Quantum Gravity **18**, 4265 (2001)
17. Maartens, R.: Living Rev. Relativ. **7**, 7 (2004)
18. Setare, M.R.: Eur. Phys. J. C **33**, 555 (2004)
19. Setare, M.R., Vagenas, E.C.: Phys. Lett. B **584**, 127 (2004)
20. Saharian, A.A., Setare, M.R.: Nucl. Phys. B **724**, 406 (2005)
21. Kiritsis, E.: J. Cosmol. Astropart. Phys. **0510**, 014 (2005)
22. Krauss, P.: J. High Energy Phys. **9912**, 011 (1999)
23. Ida, D.: J. High Energy Phys. **0009**, 014 (2000)
24. Barcelo, C., Visser, M.: Phys. Lett. B **482**, 183 (2000)
25. Stoica, H., Henry Ty, S.-H., Wasserman, I.: Phys. Lett. B **482**, 205 (2000)
26. Gomez, C., Janssen, B., Silva, P.J.: J. High Energy Phys. **0004**, 027 (2000)
27. Kamenshchik, A., Moshella, U., Pasquier, V.: Phys. Lett. B **487**, 7 (2000)
28. Bowcock, P., Charmousis, C., Gregory, R.: Class. Quantum Gravity **17**, 4745 (2000)
29. Birmingham, D., Rinaldi, M.: Mod. Phys. Lett. A **16**, 1887 (2001)
30. Csaki, C., Erlich, J., Grojean, C.: Nucl. Phys. B **604**, 312 (2001)
31. Nojiri, S., Odintsov, S.D., Ogushi, S.: Int. J. Mod. Phys. A **18**, 3395 (2003)
32. Nojiri, S., Odintsov, S.D.: Int. J. Mod. Phys. A **18**, 2001 (2002)
33. Nojiri, S., Odintsov, S.D.: Phys. Lett. B **562**, 9 (2003)
34. Verlinde, E.: hep-th/0008140 (2008)
35. Setare, M.R.: Mod. Phys. Lett. A **17**, 2089 (2002)
36. Setare, M.R., Mansouri, R.: Int. J. Mod. Phys. A **18**, 4443 (2003)
37. Setare, M.R., Altaie, M.B.: Eur. Phys. J. C **30**, 273 (2003)
38. Setare, M.R., Vagenas, E.C.: Phys. Rev. D **68**, 064014 (2003)
39. Mukherji, S., Peloso, M.: Phys. Lett. B **547**, 297 (2002)
40. Hawking, S.W., Ellis, G.F.R.: The Large Scale Structure of Spacetime. Cambridge University Press, Cambridge (1973)
41. Vollick, D.N.: Gen. Relativ. Gravit. **34**, 1 (2002)
42. Medved, A.J.: J. High Energy Phys. **0305**, 008 (2003)
43. Coule, D.H.: Class. Quantum Gravity **18**, 4265 (2001)
44. Foffa, S.: Phys. Rev. D **68**, 043511 (2003)
45. Myung, Y.S.: Class. Quantum Gravity **20**, 935 (2003)
46. Biswas, A., Mukherji, S., Pal, S.S.: Int. J. Mod. Phys. A **19**, 557 (2004)
47. Kanti, P., Tamvakis, K.: Phys. Rev. D **68**, 024014 (2003)
48. Setare, M.R.: Phys. Lett. B **602**, 1 (2004)
49. Setare, M.R.: Phys. Lett. B **612**, 100 (2005)
50. Setare, M.R.: Eur. Phys. J. C **47**, 851 (2006)
51. Hovdebo, J.L., Myers, R.C.: J. Cosmol. Astropart. Phys. **0311**, 012 (2003)
52. Kaul, R.K., Majumdar, P.: Phys. Rev. Lett. **84**, 5255 (2000)
53. Carlip, S.: Class. Quantum Gravity **17**, 4175 (2000)
54. Das, S., Majumdar, P., Bhaduri, R.K.: Class. Quantum Gravity **19**, 2355 (2002)
55. Lidsey, J.E., Nojiri, S., Odintsov, S.D., Ogushi, S.: Phys. Lett. B **544**, 337 (2002)
56. Setare, M.R.: Phys. Lett. B **573**, 173 (2003)
57. Setare, M.R.: Eur. Phys. J. C **33**, 555 (2004)
58. Kraus, P., Wilczek, F.: Nucl. Phys. B **433**, 403 (1995)

59. Keski-Vakkuri, E., Kraus, P.: Nucl. Phys. B **491**, 249 (1997)
60. Parikh, M.K., Wilczek, F.: Phys. Rev. Lett. **85**, 5042 (2000)
61. Kwon, Y.: Nuovo Cim. B **115**, 469 (2000)
62. Hemming, S., Keski-Vakkuri, E.: Phys. Rev. D **64**, 044006 (2001)
63. Setare, M.R., Vagenas, E.C.: Int. J. Mod. Phys. A **20**, 7219 (2005)
64. Cavaglia, M., Das, S.: Class. Quantum Gravity **21**, 4511 (2004)
65. Das, S.: Pramana **63**, 797 (2004)
66. Setare, M.R.: Phys. Rev. D **70**, 087501 (2004)
67. Hawking, S.W.: Commun. Math. Phys. **43**, 199 (1975)
68. Campos, A., Sopuerta, C.F.: Phys. Rev. D **63**, 104012 (2001)
69. Campos, A., Sopuerta, C.F.: Phys. Rev. D **64**, 104011 (2001)
70. Goliath, M., Ellis, G.F.R.: Phys. Rev. D **60**, 023502 (1999)
71. Iakubovskyi, D., Shtanov, Y.: Class. Quantum Gravity **22**, 2415 (2005)
72. Savonije, I., Verlinde, E.: Phys. Lett. B **305**, 507 (2001)